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CONVERGENCE OF RANDOM MEASURES AND POINT PROCESSES ON THE PLANE

BY

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Abstract. In the paper two topologies in the space of realizations of random measures on the plane are discussed. The first one is usual vague topology while another is relativized Skorohod's topology. The relation between convergence of random measure in these topologies and conditions for convergence in relativized Skorohod's topology are established.

1. Introduction. The space M of realizations of random measures on the plane may be considered as the subset of the space $D[0, \infty)^2$ of "continuous from above, with limits from below" functions. Thus the relativized Skorohod's topology is another candidate for topology on M instead of the usual vague topology.

The relation between convergence in distribution of random measures in vague topology and relativized Skorohod's topology (s-topology) is investigated. The conditions for relative compactness of a sequence of random measures w.r.t. convergence in the s-topology are given. They are simpler than the conditions of relative compactness of a sequence of general random fields in the space $D[0, \infty)^2$.

From the results obtained the conditions for the convergence of superpositions of independent point processes to the Poisson processes are derived.

2. The space $D = D[0, \infty)^2$. Let $T = [0, \infty)^2 = \{t = (t_1, t_2)\colon 0 \leqslant t_i < \infty,\ i = 1, 2\}$. For $t = (t_1, t_2) \in T$ we define

$$E_t^{11} = \{s = (s_1, s_2) \in T\colon s_1 \geqslant t_1,\ s_2 \geqslant t_2\}, \quad E_t^{00} = \{s \in T\colon s_1 < t_1,\ s_2 < t_2\},$$
$$E_t^{10} = \{s \in T\colon s_1 \geqslant t_1,\ s_2 < t_2\}, \quad E_t^{01} = \{s \in T\colon s_1 < t_1,\ s_2 \geqslant t_2\}.$$

We equip T with the usual topology, generated by the norm

$$\|t\| = \max(t_1, t_2).$$

Let $D = D[0, \infty)^2$ denote the space of real valued functions on T which are "continuous from above, with limits from below" in the following sense: for each $t \in T$

(a) $\lim x(s)(s \to t, s \in E_t^{ij})$ exists for all (i, j), $i, j = 0, 1$,

and

(b) $x(t) = \lim x(s)(s \to t, \; s \in E_t^{11})$.

Let Λ be the class of all homeomorphisms of T onto itself of the form $\lambda = (\lambda_1, \lambda_2)$ where λ_i is a strictly increasing continuous function on $[0, \infty)$ with $\lambda_i(0) = 0$, $\lambda_i(\infty) = \infty$, $i = 1, 2$, and the image of a point $t = (t_1, t_2)$ under a homeomorphism $\lambda = (\lambda_1, \lambda_2)$ is $\lambda t = (\lambda_1 t_1, \lambda_2 t_2)$.

Let $x, x_1, x_2, \ldots \in D$ be given. We say that x_n *S-converges* to x if there exist $\lambda_1, \lambda_2, \ldots \in \Lambda$ such that for each real $a > 0$

$$\lim_{n \to \infty} \sup_{\|t\| \leqslant a} |x_n(\lambda_n t) - x(t)| = 0$$

and

$$\lim_{n \to \infty} \sup_{\|t\| \leqslant a} \|\lambda_n t - t\| = 0.$$

The topology which corresponds to S-convergence we call *S-topology*. It coincides with the topology discussed in [4], and is analogous to the well-known Skorohod's topology (cf. [1], [6], [9]). With the respect to S-topology, D is separable and topologically complete, and the Borel σ-algebra $\mathfrak{D}$ coincides with the σ-algebra generated by the coordinate mappings (cf. [4]).

Given a probability measure P on $(D, \mathfrak{D})$, let T_P consist of those $t \in T$ for which

$$P\{x \in D\colon \, x \text{ is continuous at } t\} = 1.$$

For $a = (a_1, a_2)$ let $D([0, a_1] \times [0, a_2])$ denote the space of real valued functions on

$$[0, a_1] \times [0, a_2] = \{t = (t_1, t_2) \in T\colon \, t_i \leqslant a_i, \; i = 1, 2\},$$

which are "continuous from above, with limits from below". The space $D([0, a_1] \times [0, a_2])$ is analogous to $D[0, 1]^2$ (for $D[0, 1]^2$ cf. [2], [8], [9]).

Let $\tau_a\colon \, D \to D([0, a_1] \times [0, a_2])$ be defined by

$$\tau_a x(t) = x(t), \quad t_1 \leqslant a_1, \; t_2 \leqslant a_2.$$

From [4] we have

THEOREM 1. *Let $P, P_1, P_2, \ldots$ be probability measures on $(D, \mathfrak{D})$. Then $P_n \Rightarrow P$ if and only if $P_n \tau_a^{-1} \Rightarrow P \tau_a^{-1}$ for all $a \in T_p$. (Here $\Rightarrow$ means weak convergence).*

By this theorem one can restrict oneself to the space $D[0, 1]^2$ when considering the convergence in $D[0, \infty)^2$.

3. The space of realizations of random measures. Let $M = M(T)$ denote the class of all measures on $(T, \mathfrak{B}(T))$ such that $\mu(\{0\} \times B) = \mu(B \times \{0\}) = 0$, $B \in \mathfrak{B}(R_+)$ and $\mu A < \infty$ for all bounded $A \in \mathfrak{B}(T)$. (Here $\mathfrak{B}(T)$ and $\mathfrak{B}(R_+)$ mean Borel σ-algebras of subsets of T and $R_+ = [0, \infty)$, respectively). Every $\mu \in M$ can be regarded as a function in $D = D(T)$ which we call the distribution function of μ and denote also by μ. Thus, we will not distinguish between a measure and its distribution function.

Let s denote the relative topology on M. M is a closed subset of D. Thus, topological space (M, s) is separable and topologically complete. Let $\mathfrak{M}_s$ denote the Borel σ-algebra of subsets of (M, s). We have $\mathfrak{M}_s \subset \mathfrak{D}$.

Let $\mu, \mu_1, \mu_2, \ldots \in M$ be given. We say that μ_n v-converges to μ if $\mu_n(t) \to \mu(t)$ for all continuity points t of μ.

One can introduce a metric corresponding to v-convergence which makes M separable and complete (cf. [3], [5]). We denote by v the corresponding topology and by $\mathfrak{M}_v$ the Borel σ-algebra of subset of (M, v). s-convergence implies v-convergence but not vice versa. Thus $\mathfrak{M}_s \subset \mathfrak{M}_s$. Actually, $\mathfrak{M}_v$ and $\mathfrak{M}_s$ coincide with the σ-algebra generated by coordinate mappings (cf. [4], [5], [9]). Let us put $\mathfrak{M} = \mathfrak{M}_s = \mathfrak{M}_v$.

Write

$$N = \{\mu \in M \colon \mu B \in Z_+ = \{0, 1, \ldots\},\ B \in \mathfrak{B}(T)\}.$$

N is s-closed and v-closed subset of M and $N \in \mathfrak{M}$.

We turn now to the problem of characterizing the compact subsets of the topological space (M, s). By Theorem 1 we may restrict ourselves to the space of functions defined on $[0, 1]^2$.

Let $D_0 = D[0, 1]^2$ $(M_0 = M[0, 1]^2)$ be the space of functions in D (in M, respectively) restricted to $[0, 1]^2 = \{(s, t)\colon 0 \leqslant s, t \leqslant 1\}$ with Skorohod (relative) topology (cf. [2], [8], [9] for the space D_0). We may assume that all functions in M_0 are continuous at $(1, 1)$.

For $x \in M_0$, $t \in (0, 1)$ and $\delta > 0$ set

$$w_\delta^{(1)}(t, x) = \int_{(t-\delta, t+\delta)} [x(t+\delta, 1) - x(s, 1)]\, dx(s, 1)$$

and

$$w_\delta^{(2)}(t, x) = \int_{(t-\delta, t+\delta)} [x(1, t+\delta) - x(1, s)]\, dx(1, s)$$

THEOREM 2. *A set $A \subset M_0$ has compact closure in the Skorohod (relative) topology if and only if $\sup_{x \in A} x(1, 1) < \infty$ and*

$$\lim_{\delta \to 0} \sup_{x \in A} x(\delta, 1) = \lim_{\delta \to 0} \sup_{x \in A} x(1, \delta) = 0, \tag{1}$$

(2) $$\lim_{\delta\to 0}\sup_{x\in A}[x(1,1)-x(1-\delta,1)] = \lim_{\delta\to 0}\sup_{x\in A}[x(1,1)-x(1,1-\delta)] = 0$$

and, for all $t\in(0,1)$,

(3) $$\lim_{\delta\to 0}\sup_{x\in A} w_\delta^{(i)}(t,x) = 0, \quad i = 1, 2.$$

Proof. For $x\in M_0$ and $0<\delta<1$, put

$$\omega'_x(\delta) = \inf \max_{i,j} |x(s^{(i)}-0,\ t^{(j)}-0)-x(s^{(i-1)},\ t^{(j-1)})|,$$

where the infimum extends over the finite collections $\{(s^{(i)}, t^{(j)})\}$ of points $(s^{(i)}, t^{(j)})$ satisfying the following conditions:

(4) $$\begin{cases} 0 = s^{(0)} < s^{(1)} < \ldots < s^{(p)} = 1, \\ 0 = t^{(0)} < t^{(1)} < \ldots < t^{(q)} = 1, \\ s^{(i)}-s^{(i-1)} > \delta,\ t^{(j)}-t^{(j-1)} > \delta, \quad i = 1, \ldots, p;\ j = 1, \ldots, q. \end{cases}$$

If A has a compact closure, then $\sup_{x\in A} x(1,1) < \infty$ and (cf. [8])

(5) $$\lim_{\delta\to 0}\sup_{x\in A}\omega'_x(\delta) = 0.$$

Obviously, conditions (1) and (2) also hold.

Given δ and ε, decompose $[0,1)^2 = \{(s,t)\colon 0\leqslant s, t<1\}$ into rectangles $[s^{(i-1)} s^{(i)})\times[t^{(j-1)}, t^{(j)})$, such that $s^{(i)}-s^{(i-1)} > 2\delta$, $t^{(j)}-t^{(j-1)} > 2\delta$ and

$$\max_{i,j}[x(s^{(i)}-0,\ t^{(j)}-0)-x(s^{(i-1)}, t^{(j-1)})] < \omega'_x(2\delta)+\varepsilon.$$

Then for any $t\in(0,1)$ either $(t-\delta, t+\delta)\subset[s^{(i-1)}, s^{(i)})$ for some i, $1\leqslant i\leqslant p$, in which case

(6) $$x(t+\delta,1)-x(t-\delta,1) < x(s^{(i)}-0,1)-x(s^{(i-1)},1) < \omega'_x(2\delta)+\varepsilon,$$

or some $s^{(i)}$ lies in the interval $(t-\delta, t+\delta)$, in which case

(7) $$\begin{cases} x(s^{(i)}-0,1)-x(t-\delta,1) < \omega'_x(2\delta)+\varepsilon, \\ x(t+\delta,1)-x(s^{(i)},1) < \omega'_x(2\delta)+\varepsilon. \end{cases}$$

If (6) holds, we have

(8) $$w_\delta^{(1)}(t,x) \leqslant x(1,1)[x(t+\delta,1)-x(t-\delta,1)] < x(1,1)[\omega'_x(2\delta)+\varepsilon].$$

If (7) holds, we can divide the interval $(t-\delta, t+\delta)$ into subintervals $(t-\delta, s^{(i)})$ and $[s^{(i)}, t+\delta)$. Then we have

(9) $$w_\delta^{(1)}(t,x) < 2x(1,1)[\omega'_x(2\delta)+\varepsilon].$$

By (8) and (9) we have $w_\delta^{(1)}(t,x)\leqslant 2K\omega'_x(2\delta)$, where

$$K = \sup_{x\in A} x(1,1).$$

Analogously, $w_\delta^{(2)}(t, x) \leqslant 2K\omega_x'(2\delta)$. Thus, (5) implies (4) which proves the necessity.

To prove the sufficiency, define

$$\Delta_x(\delta) = \max_{i=1,2} \Delta_x^{(i)}(\delta),$$

where

$$\Delta_x^{(1)}(\delta) = \sup\{\min \|x(t, \cdot) - x(s, \cdot)\|, \|x(u, .) - x(t, \cdot)\|, s \leqslant t \leqslant u, u - s \leqslant \delta\},$$

$$\|x(t, \cdot) - x(s, \cdot)\| = \sup_u |x(t, u) - x(s, u)|;$$

the modulus $\Delta_x^{(2)}(\delta)$ being defined analogously.

It suffices to show that (cf. [2], [7])

$$\lim_{\delta \to 0} \sup_{x \in A} \Delta_x^{(i)}(\delta) = 0, \quad i = 1, 2. \tag{10}$$

For arbitrary $\delta > 0$, $t \in (0, 1)$ and $t^* \in (t-\delta, t+\delta)$, we have

$$\begin{aligned} w_\delta^{(1)}(t, x) &= x(t+\delta, 1)[x(t+\delta-0, 1) - x(t-\delta, 1)] - \\ &\quad - \int_{(t-\delta, t^*]} x(s, 1)\,dx(s, 1) - \int_{(t^*, t+\delta)} x(s, 1)\,dx(s, 1) \\ &\geqslant [x(t^*, 1) - x(t-\delta, 1)][x(t+\delta, 1) - x(t^*, 1)] \\ &\geqslant [\min\{x(t^*, 1) - x(t-\delta, 1);\ x(t+\delta, 1) - x(t^*, 1)\}]^2. \end{aligned}$$

Thus

$$\Delta_x^{(1)}(2\delta) \leqslant [\sup_t w_\delta^{(1)}(t, x)]^{1/2}. \tag{11}$$

Analogously

$$\Delta_x^{(2)}(2\delta) \leqslant [\sup_t w_\delta^{(2)}(t, x)]^{1/2}. \tag{12}$$

It is easy to see that from (3) it follows

$$\lim_{\delta \to 0} \sup_{x \in A} \sup_t w_\delta^{(i)}(t, x) = 0, \quad i = 1, 2. \tag{13}$$

In fact, if this were not true, we could find $\varepsilon > 0$, t_k and $\delta_k \to 0$ such that $w_{\delta_k}^{(i)}(t_k, x) > \varepsilon$ for some $x \in A$, $k = 1, 2, \ldots$ If $t_k \to t_0$, then for an arbitrary δ the interval $(t_0 - \delta, t_0 + \delta)$ contains $(t_k - \delta_k, t_k + \delta_k)$ for sufficiently large k, and hence

$$w_\delta^{(i)}(t_0, x) > w_{\delta_k}^{(i)}(t_k, x) > \varepsilon,$$

which contradicts (3).

Now by (11)-(13) condition (3) implies (10).

4. Convergence of random measures. By a *random measure* on T we mean any measurable mapping of some fixed probability space $(\Omega, \mathfrak{S}, P)$ into $(M, \mathfrak{M})$.

An N-valued random measure is called the *point process.*

Given a random measure ξ on T let T_ξ consist of those $t \in T$ for which $P(\xi\{t\} = 0) = 1$. Write $\bar{T}_\xi = T \setminus T_\xi$.

For a random measure ξ and $t = (t_1, t_2)$ we set

$$\xi(t) = \xi\{[0, t_1] \times [0, t_2]\}.$$

We denote by $\xi_n \xrightarrow{\text{d,s}} \xi$ or $\xi_n \xrightarrow{\text{d,v}} \xi$ convergence in distribution of random measures ξ_n to ξ in the s-topology or v-topology, respectively.

THEOREM 3. *Let $\xi, \xi_1, \xi_2, \ldots$ be random measures. Then $\xi_n \xrightarrow{\text{d,s}} \xi$ if and only if $\xi_n \xrightarrow{\text{d,v}} \xi$ and the sequence $\{\xi_n\}$ is relatively compact w.r.t. convergence in distribution in the s-topology.*

The proof follows immediately from the facts that $\xi_n \xrightarrow{\text{d,v}} \xi$ is equivalent to $(\xi_n(t_1), \ldots, \xi_n(t_k)) \xrightarrow{\text{d}} (\xi(t_1), \ldots, \xi(t_k))$ for all $k \in Z_+$ and all $t_1, \ldots, t_k \in T_\xi$, and that the distribution of any random measure ξ is determined by the distributions of $(\xi(t_1), \ldots, \xi(t_k))$, $k \in Z_+$ with $t_1, \ldots, t_k$ running over an arbitrary dense subset of T.

Given a random measure ξ on T, $a > 0$, $t > 0$ and δ, write

$$w_\delta^{(1)}(a, t, \xi) = \int_{(t-\delta, t+\delta)} [\xi(t+\delta, a) - \xi(s, a)]\, \xi(ds, a)$$

and

$$w_\delta^{(2)}(a, t, \xi) = \int_{(t-\delta, t+\delta)} [\xi(a, t+\delta) - \xi(a, s)]\, \xi(a, ds).$$

THEOREM 4. *Let $\xi, \xi_1, \xi_2, \ldots$ be random measures. Then $\xi_n \xrightarrow{\text{d,s}} \xi$ if and only if the following conditions hold:*

for all $k \in Z_+$ and $t_1, \ldots, t_k \in T_\xi$

$$(\xi_n(t_1), \ldots, \xi_n(t_k)) \xrightarrow{\text{d}} (\xi(t_1), \ldots, \xi(t_k))$$

and, for all $a > 0$, $t = (t_1, t_2) \in \bar{T}_\xi$ and $\varepsilon > 0$,

$$\lim_{\delta \to 0} \limsup_{n \to \infty} P\{w_\delta^{(i)}(a, t_i, \xi_n) > \varepsilon\} = 0. \tag{14}$$

COROLLARY. *If $T_\xi = T$, then $\xi_n \xrightarrow{\text{d,s}} \xi$ if and only if $\xi_n \xrightarrow{\text{d,v}} \xi$.*

The proof follows from Theorems 2 and 3.

5. Convergence of superpositions of point processes. Here we apply Theorem 4 to obtain conditions of convergence of superpositions of point processes to the Poisson process.

A triangular array of point processes $\xi_{n1}, \ldots, \xi_{nk_n}$ $(n = 1, 2, \ldots)$ is called *infinitesimal* if, for all $t = (t_1, t_2) \in T$,

$$\lim_{n \to \infty} \max_{1 \leqslant k \leqslant k_n} P\{\xi_{nk}(t) > 0\} = 0.$$

We recall that $\xi(t)$ denotes the number of points of ξ in the closed rectangle $[0, t_1]\times[0, t_2]$.

A point process ξ is called *Poisson process with intensity* λ, $\lambda\in M$, if $\xi A_1, \ldots, \xi A_k$, $k\in Z_+$, $A_i\in\mathfrak{B}(T)$ are independent when $A_1, \ldots, A_k$ are disjoint and ξA has the Poisson distribution with parameter λA for all bounded $A\in\mathfrak{B}(T)$.

For $\lambda\in M$ let T_λ be the set of continuity points of λ. Write $\bar{T}_\lambda = T\backslash T_\lambda$.

THEOREM 5. *Let $\xi_{n1}, \ldots, \xi_{nk_n}$ $(n\in Z_+)$ be an infinitesimal array of independent point processes on T. Let ξ be a Poisson process with intensity λ. Then*

$$\xi_n = \sum_{k=1}^{k_n} \xi_{nk} \xrightarrow{\text{d,s}} \xi \tag{15}$$

if and only if the following conditions hold:
for all $t\in T$

$$\sum_{k=1}^{k_n} P\{\xi_{nk}(t) > 1\} \to 0, \quad n\to\infty; \tag{16}$$

for all $t'\in T_\lambda$

$$\Lambda_n(t') = \sum_{k=1}^{k_n} P\{\xi_{nk}(t') > 0\} \to \lambda(t'), \quad n\Rightarrow\infty, \tag{17}$$

and for all $a>0$ and $u=(u_1, u_2)\in\bar{T}_\lambda$,

$$\lim_{\delta\to 0}\limsup_{n\to\infty} w_\delta^{(i)}(a, u_i, \Lambda_n) = 0, \quad i=1, 2, \tag{18}$$

where

$$w_\delta^{(1)}(a, u_1, \Lambda_n) = \int_{(u_1-\delta, u_1+\delta)} [\Lambda_n(u_1+\delta, a)-\Lambda_n(s, a)]\,\Lambda_n(ds, a),$$

$w_\delta^{(2)}(a, u_2, \Lambda_n)$ being defined analogously.

COROLLARY. *If λ is continuous, then $\xi_n \xrightarrow{\text{d,s}} \xi$ if and only if for all $t\in T$*

$$\lim_{n\to\infty}\sum_{k=1}^{k_n} P\{\xi_{nk}(t) > 1\} = 0$$

and

$$\lim_{n\to\infty}\sum_{k=1}^{k_n} P\{\xi_{nk}(t) > 0\} = \lambda(t).$$

Proof. By [5], $\xi_n \xrightarrow{\text{d,v}} \xi$ if and only if (16) and (17) hold. So by Theorem 4 it suffices to show that condition (14) of Theorem 4 implies (18), and vice versa.

Let $(u, v)\in\bar{T}_\lambda$ and $a>0$ be given. Write

$$\tau_{nk} = \min\{t:\ \xi_{nk}(t, a)\geqslant 1\}, \qquad k=1, \ldots, k_n.$$

We have

$$P\{w_\delta^{(1)}(a, u, \xi_n) > 0\} \geqslant P\{\bigcup_{k \neq l} (u-\delta < \tau_{nk} < \tau_{nl} \leqslant u+\delta,\ \tau_{nm} > a,\ m \neq k, l)\}$$

$$\geqslant P\{\xi_n(a, a) = 0\} \sum_{k \neq l} \int [P\{\xi_{nl}(u+\delta, a) > 0\} -$$

$$- P\{\xi_{nl}(s, a) > 0\}] \, dP\{\xi_{nk}(s, a) > 0\}$$

$$= P\{\xi_n(a, a) = 0\} \, w_\delta^{(1)}(a, u, \Lambda_n) + o(1) \quad (n \to \infty).$$

Analogously,

$$P\{w_\delta^{(2)}(a, v, \xi_n) > 0\} \geqslant P\{\xi_n(a, a) = 0\} \, w_\delta^{(2)}(a, u, \Lambda_n) + o(1).$$

Thus (14) implies (18).

Now we prove the reverse implication. We have

$$(19) \qquad P\{w_\delta^{(1)}(a, u, \xi_n\} \geqslant 1\} \leqslant P\{\bigcup_{k \neq l} (u-\delta < \tau_k < \tau_{nl} \leqslant u+\delta)\} +$$

$$+ P\{\bigcup_k (\xi_{nk}(u+\delta, a) > 1)\}$$

$$\leqslant w_\delta^{(1)}(a, u, \Lambda_n) + o(1) \quad (n \to \infty).$$

From (19) and the analogous inequality for $w_\delta^{(2)}(a, v, \xi_n)$ it follows that (18) implies (14). This proves the theorem.

By Lemma, condition (17) together with (18) means that $\Lambda_n(\cdot)$ s-converges to $\lambda(\cdot)$. Thus we can reformulate Theorem 5 in the following way:

THEOREM 6. *Let $\xi_{n1}, \ldots, \xi_{nk_n}$, $n \in Z_+$, be an infinitesimal array of independent point processes satisfying* (16). *Let ξ be a Poisson process with intensity λ. Then*

$$\xi_n = \sum_{k=1}^{k_n} \xi_{nk} \xrightarrow{d,s} \xi$$

if and only if $\Lambda_n(\cdot)$ s-converges to $\lambda(\cdot)$.

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